Journal of Experimental Psychology: Learning, Memory, and Cognition, vol 21(1)
January, 1995

METACOGNITION AND PROBLEM SOLVING: A PROCESS-ORIENTED APPROACH

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Citation:
Metacognition and problem solving: A process-oriented approach.
Berardi-Coletta, Bernadette; Buyer, Linda S.; Dominowski, Roger L.; Rellinger, Elizabeth R.
Journal of Experimental Psychology: Learning, Memory, and Cognition, Vol 21(1), Jan 1995, 205-223. http://dx.doi.org/10.1037/0278-7393.21.1.205

ABSTRACT

Four experiments were conducted to demonstrate that the positive solution transfer effects frequently found in verbalization studies are not due to verbalization *per se*, but rather to the processes invoked when subjects are asked to report reasons or explanations for solution behaviors. It was assumed that when subjects are asked to report reasons/explanations, they must engage in self-reflective intermediate processes comparable to the metacognitive processes of monitoring, evaluating, and regulating ongoing problem solving activity. The first experiment compared various verbalization instructions to a non-verbalizing control group as well as to each other. Verbalization was categorized as either simple "think aloud" reporting, Problem-focused (answering questions regarding surface features of the problem), or Process-focused (stating reasons/explanations). The second experiment was conducted to determine whether the results of the first experiment would generalize across problem types. The third experiment compared a <u>non-verbalizing</u> Process-focused group to a non-verbalizing control group. Results from all three experiments showed that subjects in the process-focused groups performed better on both training and transfer tasks than all other groups. The fourth experiment was conducted to explore qualitative differences in problem representation and strategy use between a process-focused group and a "think-aloud" control group. Results suggest that process-focused subjects develop more sophisticated problem representations as well as more complex solution strategies than do the control subjects.

INTRODUCTION

We observe, anecdotally, in our ordinary encounters with colleagues, peers, and acquaintances that individuals engaged in a difficult or complex problem solving task often "talk aloud" while trying to find the solution. Many of us have the sense that somehow, it is helpful to "talk through" a problem; that somehow this aids the process of finding the solution. Although we are not sure specifically why it helps or what the mechanism is that provides us with such assistance, the urge to think out loud is almost irresistible.

So, what happens when we engage in such an act? What is it that appears to be so helpful? Much of the problem solving research which includes verbalization as part of the paradigm has confirmed that, indeed, some kind of thinking aloud is beneficial both during practice/learning trials and on transfer of learning to other tasks (Ahlum-Heath & DiVesta, 1986; Berry, 1983; Berry and Broadbent, 1984; 1987; Chi, Bassok, Lewis, Reimann, and Glaser, 1989; Gagné and Smith, 1962; Stinesson, 1985; Wilder and Harvey, 1971). Generally, verbalization instructions in these problem solving studies require stating reasons for solution moves, choices, trials, etc. An exception was the Chi et al. (1989) study which simply asked subjects to "think aloud." Chi et al. observed that when asked to simply "think aloud" without further instructions, good problem solvers were more likely to make statements defined as self-explanations and self-monitoring than were poor problem solvers. Thus, the relationship between verbalization, or more specifically, stating reasons, and problem solving performance seems to be a robust, positive one.

Explanations of the effect are varied. Gagné and Smith (1962) were the first to posit an explanation. They explained the beneficial effects of verbalizing on solutions to the Tower-of-Hanoi problem by stating that requiring students to verbalize makes them "stop and think." Similarly, Stinesson (1985) proposes that verbalization "slows one down," while Ahlum-Heath and Divesta (1986) suggest that a control mechanism is induced. Berry (1983) and Berry and Broadbent (1984) further specify that verbalization keeps attention focused on salient features of the problem and forces concentration on critical task components. Berry and Broadbent (1987) also propose that the timing of explanations (i.e., subject vs experimenter control of presentation of explanations) is important because "temporary cognitive units" must be activated at the time they are needed, if lasting domain knowledge is to be enhanced. Additionally, Wilder and Harvey (1971) conclude that performance is linked to "verbal mediation" whether overt or covert.

These explanations all suggest that verbalization focuses attention on and enhances problem domain knowledge. That is, verbalization "slows one down" or makes one "stop and think" more carefully about salient problem features and critical task components. This is consistent with Stein, Way, Benningfield, and Hedgecough (1986) who concluded that unless critical problem features are made explicitly salient, transfer will not occur between similar but contextually different problems. Gick and Holyoak (1980) and Catrambone and Holyoak (1989) reached similar conclusions. They all found that unless subjects were told that two problems were analogous to one another, subjects did not make the connection and were unable to use information from one problem to solve the other.

Clearly, then, it is necessary, at some level, for critical features of a problem to be attended to, remembered, and applied appropriately. Verbalization seems to enhance this component of the problem-solving process. But the question of "how" verbalization accomplishes this is not fully answered. One important variable that has not been closely examined is the type of verbalization required and/or produced. Subjects in all but one of the above-mentioned studies were required to "state a reason" or tell "why" they made a particular solution attempt, trial, or move. Dominowski (1990) has proposed that the demand that one explain oneself invokes executive processes such as monitoring, planning, and attention to problem features and that this yields "more efficient performance."

This study attempted to examine more closely the demand to "explain oneself." We suggest that answering a question such as "why did you do that?" invokes a shift in attention from focusing on aspects of the problem itself to a focus on what one is doing to solve the problem. Thus, a problem solver asked to explain his/her solution moves must take him/herself out of one mode of processing -- the problem-level --to another -- the processing-level -- and observe him/herself as a problem solver. This is what is meant by the term metacognitive processing.

Hofstadter (1979) describes metacognition as a process by which "one jumps out of the system to observe the system." Kluwe (1982) describes metacognition as an active, reflective process that is explicitly and exclusively directed at one's own cognitive activity. It involves the self-monitoring, self-evaluating, and self-regulating of ongoing tasks. We might compare this to an on-line executive processor that can observe, check, and alter routines (problem-solving strategies) already in progress. It is more than knowledge about one's self as a problem solver and more than knowledge about problem solving strategies and components, which may represent metacognitive knowledge but not metacognitive processing. Indeed, the information acquired through metacognitive processing would be of the type described by Flavell (1978), e.g., knowledge about oneself, the task (problem-domain knowledge), and the strategy (procedural knowledge). But, it must be acquired through metacognitive processing if it is to be maximally useful.

We therefore hypothesize that it is the metacognitive processes, which are invoked in response to the explanation demand, that are responsible for the improved performance in verbalization studies. This is consistent with Ericcson and Simon's (1984) extensive investigation into the effects of verbalization on problem solving. They suggest that simple "think-aloud" type verbalizations will not interfere with or change performance because subjects are simply stating what is currently in their working memory. Ericcson and Simon go on to state, however, that asking for explanations requires subjects to bring into STM that which is not normally stored there. To do this, intermediate or secondary processes must be invoked. They conceptualize a Central Processor (CP) which controls and regulates all nonautomatic processes. They distinguish between procedures which articulate that which is stored in STM, and those procedures which take stored data as their input and then perform some intermediate processes on it to produce the information requested by verbalization instructions such as "state your reasons." Thus, the CP may be interpreted as switching control from the ongoing cognitive processes (which can be verbalized in a "think-aloud" fashion) to the processes (metacognitive) directed at looking at the ongoing processes. It is implied, then, that it is the "switching" of the CP from one mode of processing to another that would be responsible for a change in performance. They further state that they could not predict the way in which this "switching" would affect performance but suggest that investigating this effect would enhance understanding of these intermediate processes. We would like to offer an interpretation of the positive effects of verbalization found in the previously mentioned studies which relies not on enhancement of problem-domain knowledge to explain the effect but, rather, on the shift in processing levels (from the problem level to the metacognitive level) that enables the solver to acquire the relevant problem knowledge and understand its application in the solution of the problem. Thus, not only is the subject more likely to solve the problem efficiently, the transfer of solution knowledge from one problem to another becomes more likely.

Four experiments were performed to test the hypothesis that it is a metacognitive, process-level focus that is responsible for improved problem-solving performance. In the first study, five groups (four verbalization and one silent control) solved four practice versions of the Tower-of-Hanoi problem and then solved a fifth, more difficult transfer version of the problem. In the second study, two verbalization groups and one silent control group served to replicate the findings from Experiment 1 and show that they generalize to a qualitatively different type of problem (the card problem (Katona, 1940)). The third study also served to replicate the results of the first experiment. This time subjects were asked to <u>think about</u> but not to verbalize their responses to the experimental prompts. The purpose of this study was to demonstrate that focusing the subjects on the process-level would produce the desired effect in the absence of verbalization.

Verbalization instructions were varied across groups in an attempt to control the focus and processing levels of each group. In the first study, as mentioned above, four verbalization groups and one silent control group solved the Tower-of-Hanoi problem. In the first verbalization group (Metacognitive group), we attempted to induce metacognitive processing by asking questions designed to <u>explicitly</u> focus the subject on what s/he was doing (monitoring) and on checking the value of solution moves (evaluating).

Another verbalization group was implicitly focused on the processing-level by requiring subjects to make an "if... then..." statement prior to making each move. This group also served to replicate previous experimental manipulations of verbalization (Ahlum-Heath and Divesta, 1986; Gagné and Smith, 1962; Stinessen, 1985). We expected this group to replicate previous findings and to perform similarly to the metacognitive group because these two groups would be engaged in similar processing modes.

The third verbalization group was asked to answer questions designed to maintain their focus on surface features of the problem. The fourth verbalization group was told simply to "talk aloud" while solving the problem with no further directions as to focus. This group functioned as a verbalization control group. A fifth group, the silent control group (given no instructions except those required to describe the problem), was included for comparison.

According to Ericcson and Simon (1984), the silent and think-aloud groups should perform similarly because simply thinking aloud (i.e., reporting the normal contents of STM) should not change performance. The process-focused (or explanation-demand) groups were expected to perform similarly to one another but better than the two control groups because intermediate (metacognitive) processes would be invoked. We expected the problem-focused group to perform worse than both the process-focused and control groups since we would be artificially limiting the focus to problem features and thus preventing spontaneous shifts to process-level operations (such as those found by Chi et al., 1989), thus potentially interfering with procedural and domain knowledge integration.

Of particular interest however, is the comparison of performance between the process-focused verbalization groups and the problem-focused group, and between the processed-focused groups and the two control groups. In general, it was predicted that the process-focused groups would perform significantly better than the both the problem-focused and control groups, both during training and on a transfer task.

EXPERIMENT 1

Subjects

One hundred and nine subjects were taken from an undergraduate subject pool and randomly assigned in approximately equal numbers to each of five groups: Metacognitive, If-then, Problem-focused, Think-aloud, and Silent-control.

Materials

The Tower-of-Hanoi puzzle was made of 3 wooden pegs anchored, approx. 3" apart, in a single row into a wooden base. The six disks were made of wood. The smallest disk was approximately 1.5" in diameter. The other disks were made so that each was approximately 0.5" larger than the preceding disk. The largest disk was approximately 4" in diameter.

Problem

Each subject was asked to solve the Tower-of-Hanoi problem under the following instructions: "This is the Tower-of-Hanoi puzzle. The goal of the problem is to move the pyramid of disks from the start peg (experimenter pointed to the stack of disks) to the goal peg (experimenter pointed to the destination peg), in as few moves as possible. I will also be timing you, however, this is of secondary importance. The number of moves you make is the most important measure. There are two rules you need to follow in solving this problem. First, you can only move one disk at a time. Second, you can never place a larger disk on top of a smaller disk. If at any time you feel you would like to go back to the starting point, or any other previous state, you may do so; this will count as one move."

Procedure

Each subject was trained on the 2-, 3-, 4-, and 5-disk versions of the Tower-of-Hanoi task, and was then presented with the 6-disk version as a transfer task. Verbalization instructions were initiated with the first practice trial (2-disk version) and continued through the last practice trial (5-disk version). No instructions to verbalize were given with the 6-disk task. Any subject who failed to solve all problems in 60 minutes was dropped from the experiment. Nine subjects were excused from the experiment because they failed to solve all problems in the allotted time. Thus, the results reported below are based on a sample of 100 students (evenly divided among the five conditions).

At the start of each problem version, the smallest of the disks were used, e.g., the two smallest disks for the 2-disk version, the three smallest for the 3-disk version, etc. Thus a larger disk was always added with each new version of the task. At the start of each problem, the pyramid of disks was always on the subject's left, and the goal was always designated as the peg to the far right.

Verbalization Instructions

In addition to the above instructions, the Metacognitive and Problem-focused groups received verbalization instructions in the form of questions that had to be answered before the subject actually made each move. All verbalizations were tape-recorded during the practice problems; the recorder was turned off during the 6-disk transfer task.

The Problem-focused group was asked one of the following questions before each move:

1. What is the goal of the problem?
2. What are the rules of the problem?
3. What is the current state of the problem (where are the disks right now)?

The Metacognitive group was asked one of the following questions before each move:

1. How are you deciding which disk to move next?
2. How are you deciding where to move the next disk?
3. How do you know that this is a good move?

For both groups, the questions were rotated in a 1, 2, 3 pattern so that each one was asked an equal number of times.

The If-then group was instructed to do the following: "Before each move, I want you to tell me where you are going to move each disk, and why. Specifically, I want you to state this in an "if-then" statement, for example, 'if I move this disk to this peg, then this will happen'."

The Think-aloud group was instructed to simply "think out loud while you are solving this problem. Try to keep talking as much as you can so that I can hear what you are thinking about as you solve the problem." Subjects were prompted to "keep talking" if they were silent for more than a few seconds.

The Silent-control group was given no additional instructions.

Protocol Analysis

Coding of the protocols consisted of classifying each statement made by each subject into one of 3 levels of focus, and a more specific content category within each major level of focus. The following is a list of the categories used in coding the Tower-of-Hanoi protocols:

I. Processing Level

a. Planning - One step ahead.

b. Planning - Two (or more) steps ahead.

c. Subgoaling

(e.g., "I need to get the large one over first;" "I have to make space for the next largest one")

d. Evaluative

(e.g., "That was a mistake;" "This seems to be working")

e. Strategy development/modification

(e.g., "The thing is, you always have to make space for the next one, and then maintain the basic pyramid shape")

II. Problem Level

a. Having to do with the stated rules of the problem

b. Having to do with the stated goal of the problem

c. Having to do with the current state of the problem, either behavioral report (I'm putting C in the middle) or reporting concrete aspects of the current state of the problem

(e.g., "I'll put this here, now this goes here" or "I have A, B, and C disks on the left peg")

d. Evaluative - negative

(e.g., "This <u>problem</u> is too hard")

e. Evaluative - positive

(e.g., "This <u>problem</u> is easy to do")

III. Personal Level

a. Evaluative - negative

(e.g., "<u>I</u> can't do this" or "I must be stupid")

b. Evaluative - positive

(e.g., "I think <u>I</u> can do this" or "This is easy <u>for me</u>")

c. Process Reflection

(e.g., "<u>I</u> always get confused at this point")

d. Questioning

(e.g., "What should I do now?" or "What's my next move?")

RESULTS

To test the main hypothesis that focusing on process-level information, i.e., invoking a metacognitive processing level, is responsible for increased ability to produce problem solution transfer, analyses of variance were used to compare performance across the groups on two different measures: ratio of excess to minimum required moves and time per move. As the task becomes more complex, the minimum number of moves required to solve the problem increases. Therefore, the ratio of excess to minimum moves was used as the dependent variable rather than moves to solution to obtain a clearer picture of performance by taking the complexity of each variation of the Tower-of-Hanoi puzzle into account.

Ratio of Excess to Minimum Moves to Solution

To test the hypothesis that process-focused groups would require fewer moves to solve the transfer problem (6-disk version) than the other three groups, a univariate *F*-test compared the mean ratio of excess to minimum required moves among groups. A significant main effect of group was found: $F_{4,95}$=5.21, MS_e = 1.78, p < .001. As can be seen by examining the five means at the extreme right of Figure 1 (the transfer data), the control groups (Silent and Think-aloud) make about 2.5 error moves for each correct move, while the If-then and Metacognitive groups make about one extra move per minimum move. The Problem-focused group's performance falls between the process-focused and control groups, suggesting that focusing on the problem was of some benefit to subjects, but not as much as focusing on the process level.

Planned comparisons contrasting group means (using Bonferroni corrected alpha levels) revealed that the Metacognitive and If-then groups did not differ ($F_{1,95}$<1). When combined to form one "process-focused" group, they differed significantly from the combined control groups ($F_{1,95}$=20.3, p <.001), which did not differ from one another ($F_{1,95}$<1). The Problem-focused group was not significantly different from either the combined control or process-focused groups.

INSERT FIGURE 1 ABOUT HERE

To test the hypothesis that process-focused groups would perform better on the learning trials than the other groups, repeated measures analyses of variance compared group performance across the learning trials (n.b., only the 2- through 5-disk versions of the problem were used in these analyses). A repeated measures ANOVA revealed a main effect of task version on the mean ratio of error to minimum moves:

Wilk's λ= .19, $F_{3,93}$=131.06, MS_e = .06, $p < .001$. A significant main effect of group was also found: $F_{4,95}$= 5.31, $p <.001$. Additionally, the interaction between groups and task version was significant: Wilk's λ= .74, $F_{2,246}$=2.48, $p < .001$. Figure 1 illustrates the performance of the 5 groups across the 4 training versions of the problem. As can be seen from the graph, the proportion of errors increases as the task becomes more complicated.

Planned comparisons of the group differences showed that the process-focused groups (Metacognitive and If-then) did not differ from one another ($F_{1,95}$=2.79, ns), but were significantly different from the combined control groups ($F_{1,95}$=16.17, $p <.001$) which, again, did not differ from one another ($F_{1,95}$=2.23, ns). The Problem-focused group differed from the process-focused groups ($F_{1,95}$=8.35, $p \leq .005$) but not from the combined controls ($F_{1,95}$< 1). Thus when looking at the relative errors made by each group, there is a clear separation of the process-level groups from all others, with the process-level groups making fewer error moves than all other groups.

Also of interest is the nature of the interaction between groups and problem complexity. As is evident from the training data shown in Figure 1, the process-focused groups (Metacognitive and If-then) are making fewer error moves for each correct move and the rate at which the ratio of excess to minimum moves increases is slower than it is for the other three groups. Here, the problem-focused group looks more like the process-focused groups in terms of rate of increase of excess moves. However, as noted earlier, in terms of number of excess moves, the Problem-focused group was found to differ from the process-focused but not from the control groups. This suggests that the Problem group is making about the same number of error moves on each version of the task as the control groups, but that its rate of change is more similar to that of the process-focused groups.

Time to Solution

To separate time to solution from its dependence on the number of moves, an analysis of variance was performed comparing the mean number of seconds/move among the five groups. There was again a significant main effect of group: $F_{4,95}$=4.01, MS_e = 9.8, $p < .005$. As indicated by the transfer data shown in Figure 2 (the means at the extreme right of the graph), the Metacognitive and If-then groups tended to spend more time per move (respectively, 6.2 and 7.8 sec) than the other three groups (approx. 5 sec each). Planned comparison F-tests contrasting group means showed that the control groups did not differ from each other ($F_{1,95}$=.358, ns), nor did the process-focused groups ($F_{1,95}$=2.85, ns). The Metacognitive and If-then groups combined, however, were different from the combined control groups ($F_{1,95}$=10.15, $p < .05$), and were different from the Problem-focused group ($F_{1,95}$=7.82, $p < .01$).

INSERT FIGURE 2 ABOUT HERE

In terms of time per move, we would expect that the groups which are asked to answer questions or state reasons should take longer for each move than than either of the control groups. A repeated measures analysis of the practice data showed that, indeed, a significant main effect of group on time per move was present: $F_{4,95}$=30.15, MS_e = 43.17, $p < .001$. These data are illustrated in the training portion of Figure 2 (2- through 5-disk versions of the Tower-of-Hanoi problem). As seen in the graph, the control groups (Silent and Think-aloud) spend about the same amount of time per move on all versions of the problem. The verbalization (questioned) groups, however, spend consistently more time than the control groups. Following performance through to the 6-disk version, one can see that although the process groups spend considerably less time per move than they had previously, they continue to spend more time, on average, than the control groups. The Problem-focused group performs more similarly on 6-disk version to the control groups than to the process-focused groups, contrary to its performance of the training tasks.

These results suggest that, in general, instructions to focus on the processing level while problem solving enhance performance when one must transfer what has been learned over a series of learning or practice tasks to a similar, but more difficult or complex new task.

Protocol Analysis

Because the verbalization instructions were presumed to affect the attention and focus of the subjects, we can look at the general content of the verbal protocols as a check to see what the subjects were attending to and remarking on. A randomly chosen subset of four protocols from each group were selected for analysis. As indicated earlier, statements made by subjects were classified into three categories: process-oriented, problem-oriented, and personal-evaluation statements. Statements were classified as process-oriented if they involved planning one or two steps ahead, subgoaling, strategy evaluation, or strategy development/modification. Statements were classified as problem-oriented if they had to do with the problem rules, goal, or the current state of the problem. Statements were considered personal if they involved comments about the solver, e.g., "You must think I'm stupid." Two graduate students independently rated the protocols and interrater reliabilities (Cohen's Kappa) ranged from from +0.76 - +1.0. As seen in Table 1, process-oriented statements made up more than 60% of the process-focused subjects' verbalizations. The Problem-focused group made no process-oriented statements and process-level statements comprised only 5% of the Think-aloud group's protocols. Rather, problem-level statements made up the majority of the verbalizations of these latter two groups (respectively, 94% and 79%). The remaining statements made by these groups were of the personal evaluative type (6% for the Problem-focused group and 16% for the Think-aloud subjects). This suggests that requiring explanations results in the subjects shifting to a process-level focus far more often than they would ordinarily.

INSERT TABLE 1 ABOUT HERE

DISCUSSION

Results from this experiment demonstrate quite clearly that it is the different types of verbalizations that the subjects are asked to produce that is involved in producing the positive effects. Requiring subjects to produce verbalizations which forced them to shift their focus to the processing level (Metacognitive and If-then groups) produced superior performance as compared to the performance of the Problem-focused subjects and subjects asked simply to "think-aloud." This effect was found for both the training and transfer problems. Examination of a randomly chosen subset of the obtained verbal protocols offered some validation for this explanation of the effect. Although we can not assume that subjects were able to report everything that they thought about during their solution attempts, it is evident from the verbal protocols that subjects in the prompted-verbalization groups (Metacognitive, If-then, and Problem-focused) did indeed focus more attention on the type of information the questions were designed to elicit. It appears that only when asked to explain or give reasons for solution attempts do subjects produce processing-level statements. This suggests that the explanation-demand produced a shift of focus that normally may occur only about 5% of the time (as compared to 60% or more for the process-focused subjects).

It is this shift in focus that may therefore be responsible for the improved performance of the process-focused groups. The Metacognitive and If-then groups were asked to examine their motives and reasons for solution behaviors as well as to evaluate their moves to solution. And, they verbally expressed an awareness of the processes in which they were engaged as they solved the problems. Relative to the Silent controls, asking subjects to simply "think aloud," i.e., refraining from inducing an attention shift, did not affect performance. The Think-aloud group's performance was statistically indistinguishable from that of the Silent controls. This is consistent with Ericcson and Simon's (1984) prediction that providing verbal reports will not alter performance unless subjects are asked to provide information not readily available from short-term memory (such as motives for behavior). We can now add that explanations for behavior are not normally attended to and, if they are, the result is positive.

Not predicted was the improved performance of the Problem-focused group relative to the controls. Although not reliably superior, this group made fewer errors on both the training and transfer tasks when it was predicted that they would perform, at best, as well as the control groups. One explanation may be that focusing on the problem, specifically by keeping track of the goal, rules, and current state of the problem, may have inadvertently contributed to the use of a means-end analysis strategy. Means-end analysis requires the solver to continually evaluate the current state of the problem relative to the goal state, and select moves based on the transformation rules. Because the problem-focused questions specifically involved these three components, subjects in this group may have been able to use this strategy more easily than those subjects who were undirected (the Think-aloud and Silent controls). This is consistent with Berry and Broadbent's (1987) conclusion that attention to salient features improves performance. However, attention to salient features did not improve performance as significantly as the requirement to provide explanations for behavior.

In sum, the data from the Experiment 1 suggest that, unless prompted, subjects do not ordinarily focus on process-level information. When specifically asked to examine what they are doing and why (process-focused groups), subjects are able to solve problems more efficiently and transfer solution knowledge to more complex problems with greater success. Again, it is more than the fact that verbalizing may enhance attention to critical problem features. Although that is an important component, and produces some benefit, it is the metacognitive processing involved in producing explanations for solution procedures that is responsible for the improved performance. Thus, shifting the focus of one's attention to one's own solution process enhances learning and produces positive transfer effects, because it allows one to monitor, evaluate, regulate, and modify what one is doing to solve the problem; it allows one to track oneself through a problem situation.

Experiment 2 was designed to replicate these findings using a different type of problem, Katona's (1940) Card problem. It is typically classified as a problem of "inducing structure" and it cannot be solved via means-end or any other linearly organized strategy. It requires a more in-depth understanding of the solution for transfer effects to occur. Because the Think-aloud group performed similarly to the Silent-control group as predicted, the Think-aloud group was not included in the second experiment. Likewise, because the If-then group's performance was similar to that of the Metacognitive group, it too was eliminated from the design of Experiment 2.

EXPERIMENT 2

The Katona Card problem (Katona, 1940) was used in the second experiment. It is different from the Tower-of-Hanoi problem in that it does not belong to the class of transformation problems, but rather to a class of problems of inducing structure (Greeno, 1978). Thus, this second experiment was performed primarily to test whether the results found in the first experiment would generalize to another kind of problem. A frequent criticism of the use of transformation problems in the study of transfer effects is that this type of problem lacks any real structure and, therefore, there is nothing of substance to be transferred (Greeno, 1978). Problems of inducing structure obviously do not lack structure. We may therefore expect metacognitive processing to have an even greater effect in producing positive transfer.

Subjects

Sixty-four subjects were taken from an undergraduate subject pool and randomly assigned in approximately equal numbers to each of three groups: Metacognitive, Problem-focused, and Silent-control.

Materials

Eight playing cards, numbered Ace - 8, all spades, were used for the practice versions of the problem. Eight different playing cards, 4 red and 4 black, randomly selected from a deck, were used for the transfer task.

Problem

The card problem involves ordering a set of playing cards, such that, when the cards are dealt, they appear in a prescribed order. The practice versions consisted of sets of 4 to 8 cards that had to be ordered such that they appeared in ascending order (Ace-4, Ace-5, etc.). However, the problem is complicated by the introduction of a "dealing rule." Subjects were told that "when the cards are dealt, they will be dealt according to the following rule: The first (top) card will be dealt face up onto the table, the next card will be dealt, face down, to the bottom of the deck. The next card will be dealt face up onto the table, the next card will be placed face down on the bottom of the deck, and so on, until all the cards have been dealt face up onto the table. You are to figure out the order in which the cards have to be arranged at the outset so that as the cards are dealt, they will appear Ace, 2, 3, and so forth."

Subjects were then told that they would not be able to use the cards to figure this out, rather, they should figure it out using pencil and paper. When they believed that they had figured out an order that would work, they were to tell the experimenter the order. The experimenter would then place the cards in the specified order and then deal the cards according to the dealing rule. The subject could then see if the proposed order "worked." If the cards did not appear in order, (e.g., Ace, 2, 4, 3), subjects were given another chance to correct their ordering of the deck. Subjects were allowed as many trials as needed to correctly solve each version of the problem.

The transfer task was a similar, but more difficult version of the 8-card task. Rather than being asked to arrange the Ace-8 so that they would appear in ascending sequence, subjects had to order a set of 8 cards, 4 red and 4 black, so that the cards would appear in alternating order (i.e., red, black, red, black, etc.). The subjects were told that the card values (e.g., King, 2, etc.) were not relevant and that they should pay attention only to the colors of the cards.

Procedure

As with the Tower-of-Hanoi problem, subjects were first given 4 "simpler" versions of this task to solve. The first version required subjects to order a deck of 4 cards, Ace-4. The second version required ordering the Ace-5. The third version required ordering the Ace-6, and the final practice version required ordering all eight cards, Ace-8. Verbalization instructions were initiated with the first practice problem (4-card version) and continued through the fourth practice problem (8-card version). Then the transfer task (Red/Black version) was introduced and no verbalization instructions were given.

Performance was measured on trials to solution and time to solution. A "trial" consisted of one "ordering" of the deck. If that order did not produce the correct outcome for the cards, a new trial was begun, and the subject was allowed to propose another order for the deck. Timing was terminated at the end of a successful trial, i.e., after the cards were dealt and the outcome was correct. As with the Tower-of-Hanoi problem, all verbalization subjects were tape-recorded during the practice tasks. The recorder was turned off when the transfer task was introduced. Subjects were provided with as much paper as they needed to solve the problems. They were allowed to look at previous solution attempts/solutions during the practice tasks, but a new sheet of paper was provided when the transfer task was introduced and all previous work sheets were removed. Four subjects were unable to complete all of the problems in 60 minutes and were excused from the experiment. Thus, the results reported below are based on 60 subjects (evenly divided among the three conditions).

Verbalization Instructions

As with the Metacognitive and Problem-focused conditions in the first experiment, subjects were asked to respond, aloud, to questions asked while they worked on the practice problems. However, because this task is not structured in the same way as the Tower-of-Hanoi problem, it was not reasonable to ask a question "before each move" as this task does not involve discrete "moves to solution." Rather, it involves discovering the underlying structure of the problem. Therefore, subjects tended to approach this problem very differently from the way subjects in the first experiment approached the Tower-of-Hanoi problem. Subjects would figure out a way of conceptualizing the process of dealing the cards, write something down to help them keep track of the process, and then revise or repeat this procedure. Questions were thus asked at times which seemed least intrusive to their thought processes. These times were a) at the beginning of each trial, b) when the subject stopped writing, c) when the subject changed the order of the cards written down, d) at points when the subject spent several seconds just looking or thinking, and e) after an unsuccessful trial. In general, questions were asked at the rate of 1 per 30-60 seconds for both the Metacognitive and Problem-focused groups, but, they were only asked after the subject demonstrated the specific behaviors described above so as to refrain from inadvertently providing hints or clues.

The Problem-focused group was asked one of the following questions at appropriate interruption points, but rotated so that all were asked an equal number of times:

1. What is the goal of the problem?
2. What is the dealing rule of the problem?
3. What cards do you have in order thus far?

These questions were designed to focus the subjects on surface features of the problem (similarly to those used for the Tower-of-Hanoi in Experiment 1) without inducing attention to other aspects of the problem.

The Metacognitive group was asked one of the following questions, according to whichever seemed most appropriate at the time (according to the below described conditions):

1. What are you thinking about in terms of starting to solve this problem? (at the beginning of a trial)
2. How are you deciding on a way to work out the order for the cards (or How are you working out the order for the cards)? (as they paused in writing down their solution)
3. How did you decide that this needed to be changed? (if they changed their solution attempt, i.e. erasing, switching the order, rewriting the order)
4. How are you deciding what went wrong? (Following an unsuccessful trial.)

The Silent-control group was given no additional instructions.

Protocol Analysis

Verbal protocols were categorized according to a schema similar to that used for the first experiment. Categories for the Katona card problem were the same as for the Tower-of-Hanoi except for the subcategories under the Processing-level category. Process-level categories for the card problem consisted of the following:

I. Processing Level

a. Procedural follow-through - verbal (e.g., "Ace goes up, 2 goes down, etc.")

b. Procedural follow-through - visual (e.g., "I'm seeing how it would deal out in my mind")

c. Strategy Development/Modification (e.g., "I'm trying to see if there's a pattern in the numbers" or "I'm looking to see how it worked last time")

d. Evaluative (e.g., "Oh, The mistake is here" or "This arrangement won't work")

RESULTS

Trials to Solution

To test the hypothesis that focus on process-level information would reduce the number of trials needed to solve the transfer problem (Red/Black version), a univariate *F*-test comparing the mean number of trials to solution among the groups was conducted. This analysis revealed a significant effect of group: $F_{2,58}$=4.53, MS_e = 6.96, p <.05. The means for the transfer task are shown at the extreme right of Figure 3. Planned comparison *F*-tests contrasting group means showed that the Metacognitive group took fewer trials to solution than the Problem-focused group ($F_{1,58}$=8.94, p<.004). The difference between the Metacognitive and the Control group approached significance ($F_{1,58}$=3.42, p=.06). Evident in Figure 3, the Metacognitive group, on average, solves the Red/Black problem in less than 2 trials while the Control and Problem-focused groups require, respectively, slightly more than 3 and 4 trials. Consistent with the results from the Tower-of-Hanoi problem, the Metacognitive group tends to perform better on the transfer task than the other groups. These results are particularly interesting when viewed in light of performance across the practice tasks.

INSERT FIGURE 3 ABOUT HERE

A repeated measures analysis of variance was used to compare mean number of trials, among groups, on the practice tasks (Ace-4 through Ace-8 versions of the problem). A significant main effect of problem version was found: Wilk's λ =.67,$F_{3,52}$=8.55, MS_e = 1.98, p<.001. A significant interaction of group and problem version was also found: Wilk's λ =.73,$F_{6,104}$=2.9, $p \leq$.01. No main effect of group was found:$F_{2,54}$=2.12, ns. The practice data is also shown in Figure 3. As one can see, the number of trials needed to solve the practice problems increases most dramatically at the 5-card task. Evident in the graph is the variability of the Problem-focused and Silent-control groups across tasks, while the Metacognitive group, after solving the 5-card version, consistently requires only 1.5 trials to solution. The difference in performance becomes especially pronounced when the Red/Black problem is introduced. *Post-hoc F*-tests contrasting mean differences in performance on the 8-card and Red/Black problems showed a significant difference between the Control and Metacognitive groups: $F_{1,54}$=4.01, MS_e = 6.04, $p \leq$.05, and between the Metacognitive and Problem-focused groups: $F_{1,54}$=6.29, $p \leq$.01. Although only a marginal difference existed between the Metacognitive and Control groups when only the transfer task was considered, this analysis makes it evident that the Metacognitive group apparently views (and handles) the Red/Black problem as "more of the same," while the performance of both the Control and Problem-focused groups deteriorates as they move from the training problems to the transfer task.

Time to Solution

An analysis of variance comparing (log transformed) time to solution for the transfer task also revealed a significant effect of group: $F_{2,58}$=3.65, MS_e = .895, p<.05. Average time to solution for the Red/Black transfer task is shown at the far right of Figure 4. As found in Experiment 1, the Metacognitive group solves in less time than the other two groups. However, this difference is significant only for the contrast between the Metacognitive and Problem-focused groups: $F_{1,58}$=6.79, $p \leq .01$. No significant differences were found in an ANOVA comparing means across groups on time/trial: $F_{2,58}$<1. This suggests that the difference in time to solution is simply a function of the number of trials to solution, i.e., the Metacognitive group solves faster because they require fewer trials.

INSERT FIGURE 4 ABOUT HERE

A repeated measures ANOVA comparing mean solution times for the groups across practice tasks revealed no significant differences (see the practice data in Figure 4). However, a similar analysis using mean time/trial as the criterion variable revealed a main effect of problem version: Wilk's λ=.65, $F_{3,52}$=8.94, MS_e = 50722.14, p<.001 and a problem version by group interaction: Wilk's λ=.79, $F_{6,104}$=2.16, $p \leq .05$. No effect of group alone was found: $F_{2,54}$=2.36, ns. These data are illustrated in the practice portion of Figure 5 (Ace-4 through Ace-8 versions of the problem). From the graph, it appears that, as the problem becomes more complex, more time is spent on each trial. Evident in the graph is the nature of the interaction: the Metacognitive group is increasing the amount of time they spend per trial at a faster rate than either of the other two groups.

INSERT FIGURE 5 ABOUT HERE

DISCUSSION

Results from Experiment 2 are consistent with those from the first experiment. The Metacognitive group shows less variability than the other groups on both the learning and transfer versions of the problem. This suggests that, for the Metacognitive group, changes in surface features (number of cards and criteria for a correct solution, e.g., Ace through 8 in sequence vs. alternating Red and Black cards) do not change their perception of the problem. Furthermore, as originally predicted, the Problem-focused group performed less well than the other groups. Here it appears that limiting the subjects' attention to surface features of the problem hindered rather than helped their performance (as opposed to the results of Experiment 1). Obviously, for this problem, keeping track of the goal, rules, and current problem state does not facilitate utilization of an effective strategy as it does for a transformation problem like the Tower-of-Hanoi. Berry and Broadbent (1987) state that attending to irrelevant features should hinder performance and it appears to be true in this case. When subjects observed and reported on what they were doing to solve a problem, their performance was enhanced. When subjects observed only surface features, their performance was interfered with. Examination of the verbal protocols again confirms that the Metacognitive group focused primarily on process level information (73% of their statements were process-oriented) while the Problem-focused group attended primarily to surface information about the the problem (93% of their statements were problem-oriented). Again, as in Experiment 1, it appears that our efforts to shift the subjects' attention from one mode to another was successful based on these verbal reports. Again, we may attribute the differences in performance to the differences in processing levels.

Although the differences between the Metacognitive and Silent-control groups' performance are not as dramatic as in Experiment 1, it is clear that the Metacognitive group performs better when the transfer task is introduced. As one can see in Figure 3, the number of trials to solution increases with the introduction of the transfer task for both the Problem-focused and Silent-control groups while the Metacognitive group's performance remains stable. This indicates that the Metacognitive group has learned more about the structure of the problem solution during the learning trials and is, therefore, better able to solve the more complex transfer task. In fact, for the Metacognitive subjects, the transfer task was apparently no more complex than the training problems, i.e., they solved the Red/Black version of the problem as easily as they solved the simpler Ace/8 version. The Problem-focused and Silent-control groups however, required significantly more trials to solution for the transfer task than they did for the last practice version of the problem thus demonstrating their relative lack of understanding of the Card problem's solution. This further illustrates how the Metacognitive group, by focusing on what they were doing as they solved the problems, became less dependent on surface features of the task and apparently understood the critical components enough to transfer their learning. The other subjects however, were quite dependent on surface features and, when the new task was introduced, their performance dropped off so much that it looks as if they were given a completely unfamiliar task.

Experiment 3 was designed to further demonstrate that these results are not dependent on verbalization but rather on the type of processing engaged in by the solver. Subjects in this study were asked to "think" rather than "talk" about the answers to process-focused questions. Only the Metacognitive and Silent-control groups were included in this study. The Tower-of-Hanoi was selected in preference to the Katona Card problem because it is simpler to administer and the results from Experiment 1 were more dramatic.

EXPERIMENT 3

The Tower-of-Hanoi problem was used again in the third experiment. In this experiment, subjects were asked the same questions as the Metacognitive subjects in Experiment 1, but were instructed not to verbalize their responses. Instead, they were asked to think about their responses. Thus, this experiment was performed as a more extreme test of the hypothesis of independence of verbalization and metacognitive processing.

Subjects

40 undergraduates participated for extra credit in their classes. They were randomly assigned in equal numbers to each of two groups: Metacognitive and Control.

Materials

The Tower-of-Hanoi puzzle was again used in this experiment.

Problems

Each subject was asked to solve the 2- through 6-disk versions of the Tower-of-Hanoi problem described in Experiment 1.

Instructions

In addition to the standard instructions for this problem, prior to each move, the Metacognitive group was asked one of the following questions:

1. How are you deciding which disk to move next?
2. How are you deciding where to move the next disk?
3. How do you know this is a good move?

As in Experiment 1, the questions were rotated in a 1, 2, 3 pattern so that each one was asked an equal number of times. However, in the current experiment, subjects were asked to think about the answers to the questions and not to answer the questions aloud. To ensure that subjects were "thinking," a six-second delay (the mean time per move obtained in Experiment 1) was imposed by the experimenter after each question was asked, before the next move could be made.

The Control group received no additional instructions.

Procedure

Each subject was trained on the 2-, 3-, 4-, and 5-disk versions of the Tower-of-Hanoi task, and then, as a transfer task, was asked to solve the 6-disk version of the Tower-of-Hanoi problem. The Metacognitive group was asked the questions listed above while they worked on the 2- through 5-disk versions (practice trials) of the Tower-of-Hanoi problem. To ensure that subjects actually considered each question, a minimum six second delay was enforced between the asking of each question and the subject's next move. No questions were asked while the subjects worked on the transfer problem.

RESULTS

To test the main hypothesis that verbalization is not a necessary component of metacognitive processing, analyses of variance were used to compare performance of the metacognitive and control groups on the following measures: ratio of excess to minimum required moves, time to solution, and time per move. In essence, the following results replicate those obtained in Experiment 1.

Moves to Solution

A univariate *F*-test compared the mean proportion of excess to minimum moves on the 6-disk version of the Tower-of-Hanoi problem (transfer problem). Again, a significant main effect of group was found: $F_{1,38}$= 7.9, MS_e =1.86, p <.005. As can be seen at the extreme right of Figure 6, the Control group is making about 2.5 error moves for every correct move on the transfer problem, while the Metacognitive group is only making slightly more than 1 error move for each required move.

INSERT FIGURE 6 ABOUT HERE

A repeated measures analysis of variance comparing the mean proportion of error moves across practice tasks resulted in a significant main effect of group across problems: $F_{1,38}$= 6.9, MS_e =.63, $p < .01$. There was also a significant main effect of problem: Wilk's $\lambda = .321$, $F_{3,36} = 25.4$, MS_e =.55, $p < .01$. The training portion of Figure 6 illustrates the profile of each group's performance across the 4 practice (2- through 5-disk) versions of the Tower-of-Hanoi problem. As one can see by examination of the figure, the proportion of error moves on the practice problems steadily increases as the task becomes more complicated.

Time To Solution

Time to solution was used as a supplementary measure of performance. A univariate *F*-test was used to compare mean (log transformed) solution times between the groups on the transfer task (6-disk problem). A significant main effect of group was found: $F_{1,38}$= 4.1, MS_e =.25, $p \leq .05$. As can be seen from the transfer data in Figure 7 (6-disk problem), the Control group took approximately 35% more time to solve the transfer problem than the Metacognitive group (733 sec vs 541 sec). To further clarify the relationship between groups in terms of time to solve the problems, time was examined separately from its dependence on moves to solution by using time per move as the dependent variable. No significant difference between the Metacognitive and Control groups was found when mean seconds/move on the 6-

disk problem was used as the criterion variable in an analysis of variance: $F_{1,38}$= 1.0, ns. Time per move on the 6-disk transfer problem is shown at the extreme right of Figure 8.

INSERT FIGURES 7 AND 8 ABOUT HERE

A repeated measures analysis of the solution time data for the practice problems showed a significant main effect of problem version on mean (log transformed) time to solution: Wilk's λ = .034, $F_{3,36}$ = 342.1, MS_e = .24, $p < .01$. Contrasting mean solution times across the training problems showed that, as expected, questioning slowed down the Metacognitive subjects: $F_{1,38}$= 62.1, MS_e = .51, $p \leq .001$. As can be seen from the training data in Figure 7 (2- through 5-disk versions of the problem), the mean solution time of the Metacognitive group exceeds that of the Control group at each training level of complexity. Thus, it is again clear that requiring subjects to stop and think about the answer to a question before each move increased time to solution. Also shown in Figure 7 is the expected trend of the Metacognitive group spending increasingly more time than the Control group to solve the successively more complex versions of the training problems. Again, consideration of this finding in light of the transfer data is illuminating. Looking at Figure 7, we see that the Metacognitive group requires no more time to solve the 6-disk version, despite the increased complexity, than the 5-disk version, while the Control group's mean time to solution has increased dramatically.

As expected an additional repeated measures ANOVA found a significant effect of group on time per move across practice tasks: $F_{1,38}$= 79.6, MS_e = 15.59, $p \leq .001$. As shown in Figure 8, during the learning trials the Metacognitive group consistently spent more time on each move than the Control group, however, this trend changed precipitously when the Metacognitive group was no longer questioned (6-disk problem).

DISCUSSION

These results clearly indicate that it is the shift in processing engaged in by the solvers (in response to explanation demand questions) which is affecting performance and not the enhanced problem-knowledge effects of verbalizing. Not only did the Metacognitive group significantly outperform the Control group, their performance is virtually indistinguishable from that of the verbalizing Metacognitive group's performance in the first experiment. While there are no verbal protocols available to determine whether the Metacognitive group was indeed focusing on process level information, it seems unreasonable to assume otherwise (in light of the protocol analysis from Experiments 1 and 2). Wilder and Harvey (1971) reached similar conclusions, i.e., overt and covert verbalizations are equally beneficial. However, we would go beyond the "verbal mediation" explanation they offered to say that it is not that subjects are "talking," whether overtly or covertly, it is what they are talking or thinking about that is beneficial. Thus, once again, we would like to conclude that metacognitive processing while problem solving results in superior performance on both learning and transfer tasks.

EXPERIMENT 4

The Katona Card problem (Katona, 1940) was used again in the fourth experiment. Because there appears to be strong, positive effect of the metacognitive-processing instructions, the present experiment was performed primarily to explore any process/strategy differences between metacognitive and talk-aloud subjects. We choose to conduct this exploration using the card problem because we felt that there were more possible approaches, relative to the Tower-of-Hanoi, that subjects might adopt in trying to solve the problem.

Subjects

Fifteen subjects were taken from an undergraduate subject pool and randomly assigned in approximately equal numbers to each of two groups: Metacognitive, and Think-aloud.

Materials

Eight playing cards, numbered Ace - 8, all spades, were used for the practice versions of the problem. Eight different playing cards, 4 red and 4 black, randomly selected from a deck, were used for the transfer task.

Problem

The five card problems were the same as those described in Experiment 2.

Procedure

The procedure was the same at that described in Experiment 2.

Performance measures included the two described in Experiment 2: trials and time to solution. As with the three earlier experiments, all verbalization subjects were tape-recorded during the practice tasks. The recorder was turned off when the transfer task was introduced. Subjects were provided with as much paper as they needed to solve the problems. They were allowed to look at previous solution attempts/solutions during the practice tasks, but a new sheet of paper was provided when the transfer task was introduced and all previous work sheets were removed. In addition to the measures mentioned above, the subjects' attempts to solve the problem were monitored more closely than in the previous experiments. For example, subjects were not allowed to erase their work, instead, every time they wanted to change the sequence in which the cards were to be stacked, they were required to explicitly write out the entire sequence. The experimenter also numbered multiple attempts to solve a single version of the problem. In this way, there could be no question about the sequence of attempts made. The experimenters also took notes describing the gross characteristics of the subjects' strategies. For example, the experimenters were told to note whether the subjects "filled in the blanks prematurely," i.e., indicated the face value of a card which would be dealt to the bottom of the deck before the subject could actually have determined which card it was. We relaxed the requirement that subjects had to complete all of the problems in 60 minutes or be excused from the experiment. Thus, the results reported below are based on all 15 subjects.

Verbalization Instructions

The verbalization instructions for the Metacognitive group were identical to those used in Experiment 2. The Metacognitive group was asked one of the following questions, according to whichever seemed most appropriate at the time (according to the below described conditions):

1. What are you thinking about in terms of starting to solve this problem? (at the beginning of a trial)
2. How are you deciding on a way to work out the order for the cards (or How are you working out the order for the cards)? (as they paused in writing down their solution)
3. How did you decide that this needed to be changed? (if they changed their solution attempt, i.e. erasing, switching the order, rewriting the order)
4. How are you deciding what went wrong? (Following an unsuccessful trial.)

The Think-aloud group was instructed to simply "think out loud while you are solving this problem. Try to keep talking as much as you can so that I can hear what you are thinking about as you solve the problem." Subjects were prompted to "keep talking" if they were silent for more than a few seconds.

RESULTS

Trials to Solution

Analyses of trials to solution were conducted to determine whether we had succeeded in replicating our earlier results with the card problem. The results of a univariate *F*-test demonstrated that significantly more ($F_{1,13}$=9.24, MS_e = .13, *p*=.01) of the Metacognitive subjects solved the Red/Black transfer version of the problem than did the Think-aloud subjects (100% and 43%, respectively, solved the transfer problem). Another univariate *F*-test was used to compare the mean number of moves required by the solvers to solve the Red/Black transfer problem. This analysis revealed a significant effect of group: $F_{1,9}$=26.73, MS_e = 1.78, *p* <.01. The means for the transfer task are shown at the extreme right of Figure 9. Evident in Figure 9, the Metacognitive group, on average, solves the Red/Black problem in approximately 1.5 trials while the solvers from the Think-aloud group required approximately 5 trials. Consistent with the results from the earlier experiments, the Metacognitive group performs better on the transfer task than the control group (both in terms of ability to solve at all and in trials to solution given the ability to solve).

INSERT FIGURE 9 ABOUT HERE

A repeated measures analysis of variance was used to compare mean number of trials, between groups, on the practice tasks (Ace-4 through Ace-8 versions of the problem). A significant main effect of problem version was found: Wilk's λ =.39,$F_{3,11}$=5.7, MS_e = 2.6, *p*=.01. A significant interaction of group and problem version was also found: Wilk's λ =.46,$F_{3,11}$=4.4, *p*<.05. There was also a main effect of group:$F_{1,13}$=10.4, MS_e = 5.8, *p*=.01. The practice data is also shown in Figure 9. As one can see, the number of trials needed to solve the practice problems increases most dramatically at the 6-card task. Evident again is the variability of the control group across tasks, while the Metacognitive group, after solving the 5-card version, consistently requires only 1.5 trials to solution. As in Experiment 2, the difference in performance becomes especially pronounced when the Red/Black problem is introduced. The performance of the Metacognitive group doesn't change, while the performance of both the Control and Problem-focused groups deteriorates.

Time to Solution

None of the analyses of time to solution were significant (all *F's* < 1.65). Although none of the differences was significant, the data in Figure 10 show that the Think-aloud group requires approximately twice as much time as the Metacognitive group to arrive at a solution for the three most difficult problems even though the Metacognitive group required more time to solve the 4- and 5-card versions of the problem .

INSERT FIGURE 10 ABOUT HERE

Strategic Differences

As in the three previous experiments, we again found the performance of the Metacognitive subjects to be superior to that of the Control subjects on the quantitative measures. Prior to discussing the observed strategic differences between the groups, a description of the various solution strategies/ representations will be given. The Katona card problem can be quite simple or extremely complex depending on the parameters of the problem and the representation/strategy the subject employs. Depending on the strategy employed by a subject, the number of cards involved and the order in which the cards are to be dealt can both affect the level of difficulty significantly. The four training problems (Ace-4, Ace-5, Ace-6, Ace-8) all involve finding a way to stack the cards such that, when they are dealt according to the every-other-card dealing rule, they will appear in sequential order. The transfer problem (Red-Black) involves stacking the cards so that they will alternate Red and Black (without regard to face value) when dealt according to the dealing rule.

The solution behaviors of the subjects were classified into five different categories: guessing (random permutation), "swapping" incorrect cards after the experimenter had dealt out a proposed solution, differential representation of cards which are dealt "up" and those dealt to the bottom of the deck, leaving blanks to indicate the position of unknown cards, and "tracing" or otherwise checking a proposed solution <u>before</u> asking the experimenter to deal the cards. Guessing (or simple permutation of the cards) is reasonably effective when the problem involves producing a sequential ordering of four cards. As the number of cards increases, guessing/permutation becomes less and less efficient and requires (on average) an increasing number of trials to solution. No subject in this experiment successfully solved the transfer problem (Red-Black) using a guessing strategy.

"Swapping" refers to a method of correcting the order of the cards by switching the cards that are out of order after the deck has been dealt. For example, a common first attempt to solve the Ace-4 version of the problem is to propose the stack be ordered Ace, 4, 2, 3. When this stack is dealt using the every-other-card dealing rule, the cards come up Ace, 2, 4, 3. A subject who has adopted a swapping strategy will note this error and switch the position of the 3 and the 4 in the stack so that when the cards are dealt again, the correct Ace, 2, 3, 4 sequence results. Because all of the training problems involve dealing the cards sequentially, a swapping strategy is reasonably effective for these problems. It works extremely well for the simpler versions of the training problems (i.e., Ace-4 and Ace-5) because, when subjects made errors, they rarely had more than two cards out place in the proposed stack. Thus, for these problems, subjects only needed one "feedback" trial to arrive at the correct sequence. Although the swapping strategy still works as the training problems become more difficult (i.e., Ace-6 and Ace-8), it becomes substantially less efficient because subjects require multiple "feedback" trials in order to correct their errors. As with the guessing strategy, the swapping strategy was likely to result in a failure to solve the transfer problem (in which face value is irrelevant to the solution) because subjects couldn't figure out where the out-of-order cards in the dealing sequence came from in the proposed stack. For example, a common first attempt at solving the transfer problem was to try interleaving two alternating sequences of cards in the initial stack, i.e., Red, red, black, black, red, red, black, black. This produces the sequence Red, black, red, black, red, red, black, black when the cards are dealt. Subjects utilizing a swapping strategy often commented "okay, so the first five in the stack are correct" and proceeded to rearrange the last three cards in the stack (positions 6, 7, and 8). Of course, the cards which are out of order in the dealt sequence came from the 4th, 6th, and 8th positions in the initial stack so that this rearrangement was usually detrimental.

One of the requirements for efficient solution of the more complex versions of the problem (Ace-6, Ace-8, Red-Black) is differential representation of the cards being dealt "up" vs. those being dealt to the bottom of the deck. In their first attempts to solve each version of the problem, subjects rarely made errors in determining which cards would be dealt "up" on the first pass through the deck. However, if they didn't differentially represent cards which were to be dealt "up" and those dealt "down," they often ended up altering the position in the stack of cards which were correctly placed. Indicating the "up" and "down" sequencing of the cards requires an appreciation for the dealing process: an understanding of how the dealing rule acts upon the proposed stack of cards to produce the observed sequence which results. This becomes increasingly important as the number of cards in the problem increases.

Using blank spaces to represent unknown cards between the known cards (the cards being dealt "up") also reflects an understanding of the way the dealing rule affects the initial positioning of the cards in the stack. This representation becomes very important for efficient solution of the more complex versions of the problem. Subjects either identified the cards in the proposed stack in sequential order, i.e., 1st, 2nd, 3rd, etc., or in the dealing order, i.e., 1st, 3rd, 5th, etc. For the simplest version of the problem (Ace-4), identifying the cards in sequential order appears to be reasonably efficient. This is perhaps due to the ability of subjects to maintain all of the problem information in working memory simultaneously, i.e., the Ace and the two will be dealt up on the first pass through the deck while the three and the four will be dealt to the bottom. However, as problem difficulty increased, subjects who employed sequential identification strategies were increasing likely to place a card which would be dealt "up" late in the first pass through the deck in a "down" position earlier in the sequence. For example, although the four in the

Ace-8 version should correctly be placed in the seventh position in the stack (i.e., Ace _ 2 _ 3 _ 4 _) a subject employing sequential identification might propose placing the four in the second position (i.e., Ace, 4, 2, ...) which "uses up" the value required for the seventh position later in the sequence. A more complex variant of the dealing-order representation used by some subjects involved the explicit representation of the substack(s) of cards remaining after some of the cards have been dealt. These subjects produced variants of the diagram shown in Figure 11 below.

INSERT FIGURE 11 ABOUT HERE

Finally, in conjunction with one of the abovementioned representations/strategies, some subjects employed a "checking" or "tracing back" strategy in which they "dealt" the stack (using paper and pencil) they were about to propose. This served to alert these subjects to assignment errors and allowed them to correct mistakes without asking the experimenter to provide a "feedback" trial by dealing the cards. Thus, these subjects could solve the problem in a single trial (as measured by requests to the experimenter to deal out the proposed solution). This checking strategy is critically important to efficient solution of the transfer problem. As stated above, because of the difficulty in identifying where red and black cards dealt out-of-order originated in the stack, the transfer problem is almost insoluble if one relies on a swapping or guessing strategy. Because the checking strategy allows subjects to work "forward" from the proposed stack to the dealing outcome (instead of working "backward" from the resulting sequence to the order which produced it), it eliminates the major difficulty subjects experience in solving the Red-Black version of the problem.

We found quite clear differences between the strategies/representations employed by the Metacognitive and Control subjects. All Control subjects utilized either guessing or swapping strategies for all versions of the problem. They either exhibited rigid adherence to the swapping strategy alone or they used it in combination with a "permutation" strategy where the subject seemed to be (and in some cases explicitly reported) simply guessing and trying, at random, configurations of the cards they had not yet tried. None of the Control subjects adopted the more sophisticated checking strategy, no matter how difficult the problems became (or how inefficient their solution strategy appeared to be). Only five of the Control subjects (71%) recognized the value of representing unknown cards by leaving blank spaces. Two of the five recognized this immediately, two more recognized it during the Ace-6 version of the problem, and one during the Ace-8 version. And, only three Control subjects (43%) represented the difference between the "up" and "down" cards: one immediately, one during the last practice trial, one during the Red/Black (transfer) version.

Metacognitive subjects, by contrast, showed greater flexibility and more appreciation for the interaction of strategies/representations with problem parameters. Only four subjects (50%) ever used the swapping strategy and three of them abandoned it early in training (by the Ace-6 version of the problem). The only Metacognitive subject who continued to use the swapping strategy for all of the training problems abandoned it when confronted with the nonsequential, alternating Red/Black version. All eight of the Metacognitive subjects developed a "tracking" or checking strategy by the time they had solved the second training problem. All of the Metacognitive subjects recognized the need to use blanks to represent cards dealt to the bottom of the deck: Five during the first training problem, one during the second problem, and the remaining two during the fourth training problem. All of the Metacognitive subjects recognized the need to differentially represent the cards dealt "up" and those dealt "under:"

DISCUSSION

The differences between the two groups in strategy use and choice of representations is striking. Most compelling is the fact that none of the Control subjects developed the sophisticated checking strategy (one subject stated, "but there's no way to tell how this will come out.") while every Metacognitive subject not only did so, but did so very early in the practice trials. This difference in strategy development suggests that only when subjects are asked to explain what they are doing and why do they ask themselves these questions. It appears that our questions led the Metacognitive subjects to recognize that they could apply the dealing process to the proposed stack and thus obtain a relatively accurate picture of what the dealing outcome would be. This, in turn, led to the Metacognitive subjects identifying their errors without having to see the cards be dealt. In addition, they utilized problem representations that were far more efficient in terms of specifying the relations of the cards in the stack to the cards in the dealt sequences. In fact, problem representation and the checking strategy are interrelated: as one tries to check the ongoing solution attempt, more complex, efficient representations of the problem are necessarily created. This in turn promotes more effective monitoring of the solution activity. The Control subjects, who were not asked to reflect on their problem solving, apparently did not do so. They relied on the cards, the experimenter, and in essence, their environment to tell them if they were right or wrong, close to or far from the solution. Control subjects required more trials for each version of the problem because completed trials provided their only feedback. Metacognitive subjects received continual feedback as they traced (on paper) the sequence from proposed stack, through application of the dealing rule, to the subsequent outcome. This resulted in the creation of more sophisticated problem representations and in the modification of their solutions as they constructed them.

GENERAL DISCUSSION

Results from the current experiments demonstrate two basic findings with regard to problem solving and solution transfer. First, subjects do not spontaneously focus on the process by which they attain a problem solution, and second, if they are forced to do so, transfer effects are positive. Other studies, recognizing that transfer must be related to knowledge of the problem solution, have tried to obtain positive transfer by looking for ways to provide subjects with solution knowledge. Neither explanations of solutions, hints, nor demonstrations of solutions produced the desired effect (Berry, 1983; Buyer, 1990; Buyer and Dominowski, 1989). The current experiments suggest that information regarding the problem solution must be acquired or discovered by the subject via metacognitive processing. When an understanding of the process by which a problem gets solved is attained in this way, positive transfer effects are found.

Initial analysis of the verbal protocols indicated that those subjects who were asked to explain what they were doing tended to talk about their strategies, their errors, and their expectations regarding the consequences of future moves, that is, they focused on process-oriented information. Note that it is not the looking ahead or strategy development which is metacognitive. Subjects in the Think-aloud, Silent, or Problem-focused groups may very well have engaged in some "looking ahead" or strategy development but they were apparently not consciously aware of doing so. If they were, they might have reported being engaged in these activities more frequently. The requirement to focus on, attend to, and become consciously aware of these solution components elicited the metacognitive processes. Kluwe (1982) states that metacognition is a process, that it is "cognitive activity directed at one's own cognitive enterprise," and not knowledge about solution strategies or problem knowledge. Metacognitive processes monitor solution strategies, regulate the stream of solution activity, and evaluate the effects of the solution process. This is precisely what we asked subjects to do by requiring them to answer (or, in Experiment 3, think about) questions pertaining to "how" they were deciding what to do next. The fact that they reported such process-oriented thoughts as planning ahead, subgoaling, and strategy development, means that we successfully engaged them in metacognitive processing. That is, when one is asked to monitor, evaluate, and regulate one's own cognitive activity, one reports such things as the strategy one is using, how well it is working, what one is attempting to do at the time, and predicting what might happen if a certain "move" is executed. Thus, the content of the verbal reports is an indicator of the level of processing in which the subject is engaged. By focusing on a process level, subjects were able to attend to information regarding the way in which they solved a problem (process-level information). The groups who had this opportunity clearly performed better. Our finding that subjects not asked to focus on "why" rarely reported any process-level information and tended to report only problem- and personal-level data indicates that attending to process-level information represents a shift from the usual realm of focus, from the problem-level to the metacognitive-level. This would explain more fully the enhanced performance of verbalization groups in other studies as well.

We also see in the verbal protocols that the process-focused subjects attended to all three levels of information (process, problem, and personal) while the Problem-focused and Think-aloud subjects tended to report only problem- and personal-level information. We know that the Metacognitive subjects reported a wider variety of statements indicating their use of a broader data base. It may be then that metacognitive processing enhances integration of procedural and domain knowledge.

A more detailed analysis of the protocols reveals additional differences that may delineate other important aspects of the "metacognitive shift." The If-Then and Metacognitive groups performed best and they tended to produce the most process-level statements. Yet, there is a difference between these groups also, with the Metacognitive group tending to perform better than the If-Then group. Correspondingly, there are differences in the type and number of processing categories used by each group. The If-Then group tended to make about the same percentage of processing-level statements (68%) as the Metacognitive group (61%). However, most of their process-level statements were classified into one of two subcategories: One-step look ahead (85%) and two-step look ahead (12%). The Metacognitive group, on the other hand used all five process-level subcategories, including subgoaling, strategy development, and evaluative statements. Further, the distribution of statements into the subcategories was more uniform for the Metacognitive than for the If-Then subjects. Table 2 shows the breakdown of the process-level statements for these two groups.

INSERT TABLE 2 ABOUT HERE

Also, as stated earlier, the Think-aloud and Problem-focused groups made virtually no process-level statements (refer to Table 1). Instead they concentrated mostly at the problem level, with the remaining statements falling into the personal-level category. Although both groups focused primarily on the problem level, the Problem-focused group out performed the Think-aloud group. When the Think-aloud subject's protocols were examined, we found that these subjects focused on at least two levels fairly consistently, the most frequent being the problem level, the other being the personal level. What we do not find in the problem-focused group is the presence of many personal-level statements. The Think-aloud group often evaluated themselves during the solution process (as opposed to evaluating their progress or the state of the problem). When the personal-level statements of this group were examined, we found that approximately 49% of the statements were classified as negative self-evaluations. Examination of the Metacognitive protocols showed a comparable percentage of personal statements, however, only 16% were negative self-evaluations while 68% were personal process statements. Table 3 shows the breakdown of relevant personal-level subcategories for these two groups.

INSERT TABLE 3 ABOUT HERE

The presence of negative personal statements suggests that one reason that the Problem-focused subjects performed better than the control subjects, may be, in part, that they do not become distracted by their own failings. They remain focused on the task. Therefore, the Problem-focused group may have performed better than the controls because they were not distracted from the task by negative thoughts about themselves as problem solvers. Additionally, it appears that metacognitive processing enables the use of self-observation without the hindrance of negative self-evaluation.

The shift, then, to a metacognitive processing level, may more fully explain results from other verbalization studies. Virtually all previous verbalization studies have relied on "reason giving" as the primary mode of verbalization. Explanations of the effect range from "it makes subjects stop and think" (Gagné and Smith, 1962) to enabling rehearsal and attention to critical features (Berry and Broadbent, 1984; 1987). However, rehearsing critical features, as did the Problem-focused groups in Experiments 1 and 2 was only minimally beneficial compared to the Metacognitive processing group. We indeed believe that this is a necessary component of problem solving, and we do believe that questions such as "why?" make one stop and think. But we also believe that the effect of explaining oneself goes beyond focussing attention on the problem *per se* and beyond verbal mediation: the requirement to explain oneself shifts the focus of the problem solver to the solver's actions, thoughts, and reasoning, and asks the solver to examine them.

Observations of subjects' strategies in Experiment 4 point clearly to the positive value of metacognitive processing. Evident in this experiment was the striking contrast in the way the Metacognitive and Control subjects approached the tasks. Metacognitive subjects switched from simple to more complex strategies, monitored themselves and the problem solution more often, and developed more sophisticated representations of the problem structure. Control subjects showed no signs of self-monitoring, relying, rather, on environmental feedback. They rigidly used simple strategies that did not involve checking or tracking the movement of the cards. It appears that the Metacognitive subjects had become active participants in their own processing and understanding of the problem. In contrast, the Control subjects acted on the problem but did not participate in constructing an understanding of the solution. They passively waited for the cards to be dealt to see if they were right or wrong, and then (if wrong) tried again with no attempt to explore what had happened. Metacognitive processing therefore seems to encourage a proactive, self-reliant discovery process that does not appear to be part of the average problem solver's repertoire.

In this way, critical components, relevant features, underlying structure, connections between structurally similar but contextually different problems, may be discovered. This knowledge that is acquired may or may not be verbalizable or reportable in any concrete, organized way, at least immediately, but it is useable and transferable, as demonstrated here. In fact, we do not know what domain knowledge the subjects in the Metacognitive group have acquired and, in a sense, this appears not to be the critical issue. The issue we have investigated is the type of processing that is being performed and not the type of knowledge being processed. It is likely that if one processes information adequately, acquisition of new information follows. As cited earlier, Chi et al. (1989) found that good problem solvers could be distinguished from poor problems solvers by the number of spontaneous self-monitoring and evaluative statements they made. Chi et al. also found that good solvers improved their knowledge base as a consequence of their monitoring behavior. This would also be consistent with Berry and Broadbent's conclusion that the timing of explanations is crucial. Obviously when explanations are sought and provided as a consequence of self-monitoring or self-evaluation those explanations will be perfectly timed and optimally useful because the right "cognitive units" will always be activated (Berry and Broadbent, 1984).

It seems then, based on the findings from these studies, that broad-based problem solving skills such as "learning how to learn" and self-observation, i.e., becoming aware of what one is doing and why need to be emphasized when problem solving skills, in any domain, are being trained. This implies that problem solving, in general, has to be viewed in terms of processing skills, rather than emphasizing the content of one's knowledge base as the critical aspect. Obviously, stored knowledge is important, without data the processes would have nothing to operate on. But, the acquisition of that knowledge base is a process that must be focused on more seriously. In sum, the emphasis in problem solving research may need to shift from focusing on human problem solvers as "memory banks" that are either sufficient (or deficient) for problem solving due to the amount of knowledge they contain, to humans as dynamic, information processors that are continually acquiring data in more or less efficient ways, the efficiency being determined largely by the presence or absence of metacognitive processing.

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TABLE 1

CATEGORY PERCENTAGES FOR EACH GROUP

	Metacognitive	If-Then	Problem	Think-aloud
Process-focused	61%	68%	0%	5%
Problem-focused	24%	27%	94%	79%
Personally-focused	14%	5%	6%	16%

TABLE 2

DATA FROM EXPERIMENT 1

PROCESS-LEVEL CATEGORY PERCENTAGES

	Metacognitive	If-Then
1-step	19%	85%
2-step	18%	12%
Subgoaling	36%	1.5%
Evaluative	13%	1%
Strategy	14%	0%
	100%	100%

TABLE 3

DATA FROM EXPERIMENT 1

PERSONAL-LEVEL CATEGORY PERCENTAGES

	Metacognitive	Think-aloud
Negative evaluation.	16%	49%
Positive evaluation.	5%	6%
Process	68%	27%
Question	11%	17%
	100%	100%

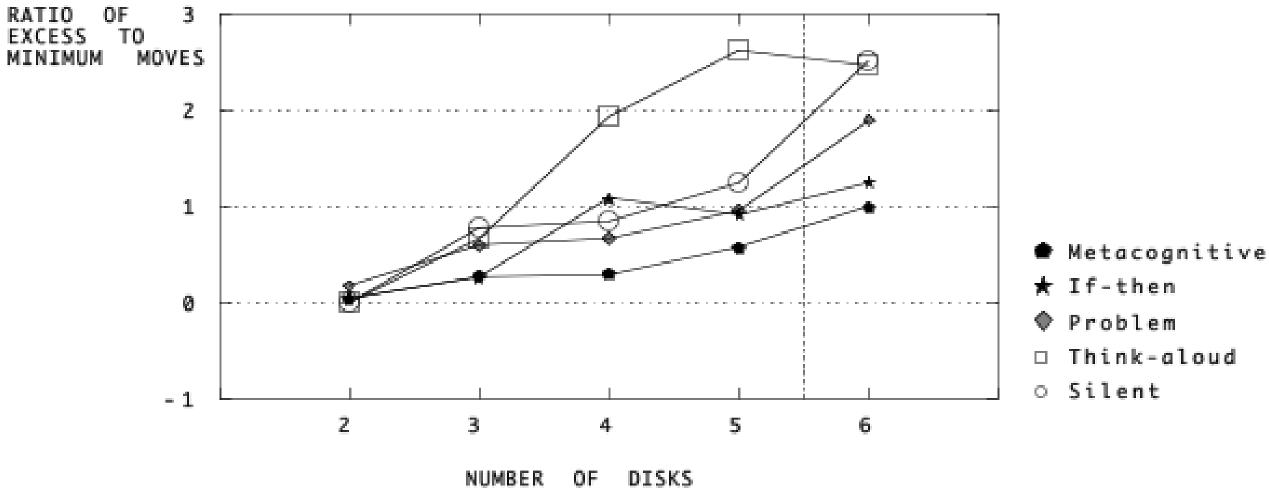

FIGURE 1: PERFORMANCE ACROSS THE 2- THROUGH 6-DISK VERSIONS OF THE TOWER-OF-HANOI PROBLEM: Mean ratio excess to minimum moves

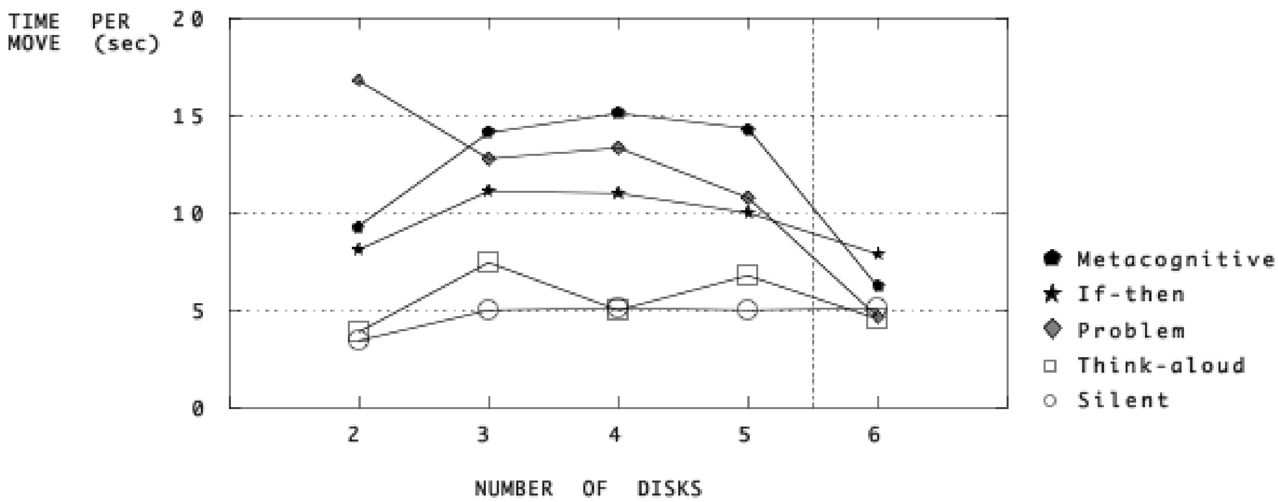

FIGURE 2: PERFORMANCE ACROSS THE 2- THROUGH 6-DISK VERSIONS OF THE TOWER-OF-HANOI PROBLEM: Mean time (sec) per move

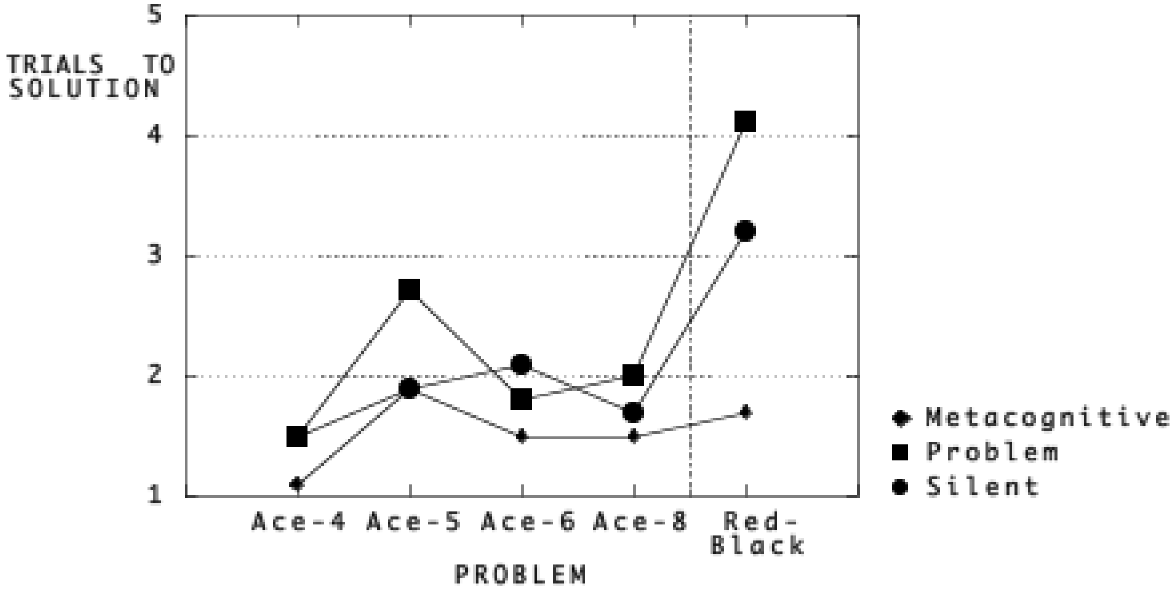

FIGURE 3: PERFORMANCE ACROSS THE 4 THROUGH 8 CARD VARIATIONS OF THE ACE, 2, 3, ... VERSION OF THE KATONA CARD PROBLEM AND ON THE RED/BLACK VERSION OF THE PROBLEM: Mean trials to solution

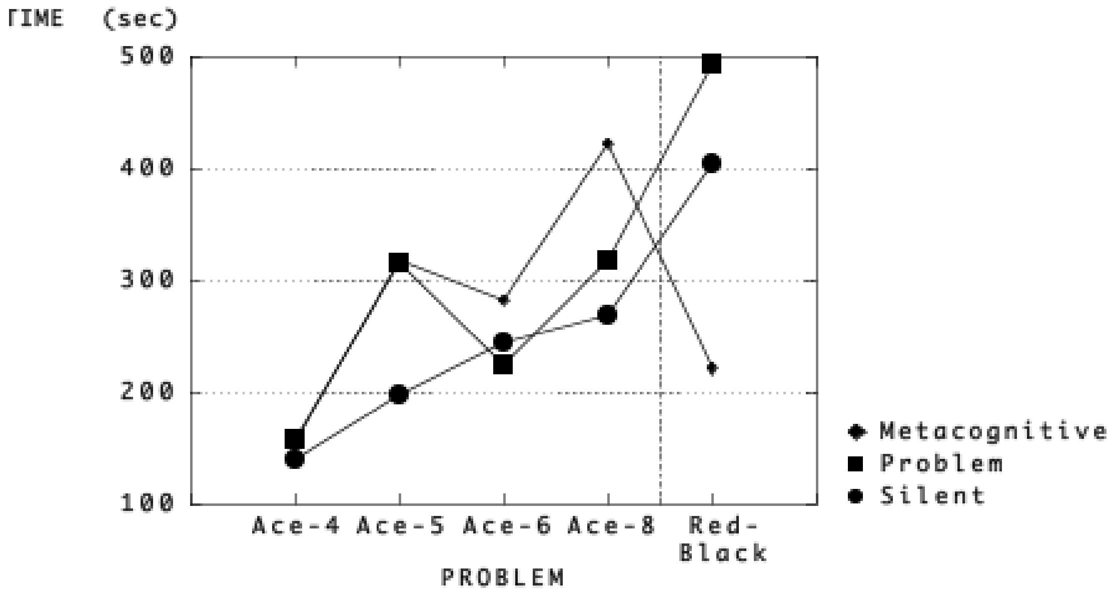

FIGURE 4: PERFORMANCE ACROSS THE 4 THROUGH 8 CARD VARIATIONS OF THE ACE, 2, 3, ... VERSION OF THE KATONA CARD PROBLEM AND ON THE RED/BLACK VERSION OF THE PROBLEM: Mean time (sec) per trial

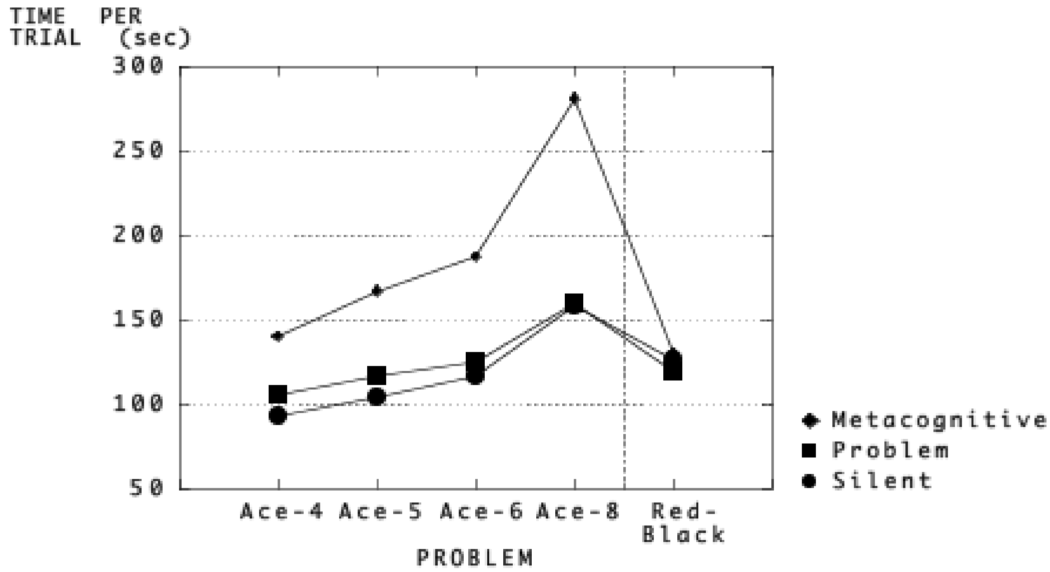

FIGURE 5: PERFORMANCE ACROSS THE 4 THROUGH 8 CARD VARIATIONS OF THE ACE, 2, 3, ... VERSION OF THE KATONA CARD PROBLEM AND ON THE RED/BLACK VERSION OF THE PROBLEM: Mean time (sec) per trial

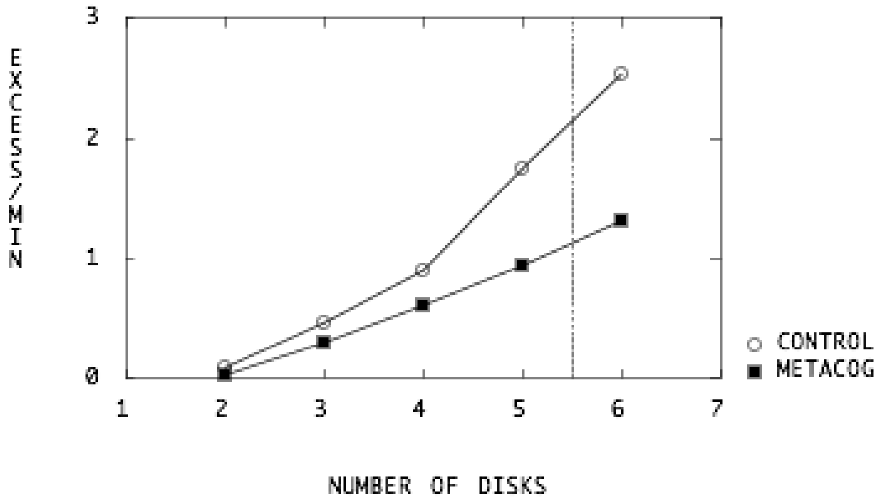

FIGURE 6: PERFORMANCE ACROSS 2- THROUGH 6-DISK VERSIONS OF THE TOWER-OF-HANOI PROBLEM (Ratio of Excess Moves to Minimum Moves)

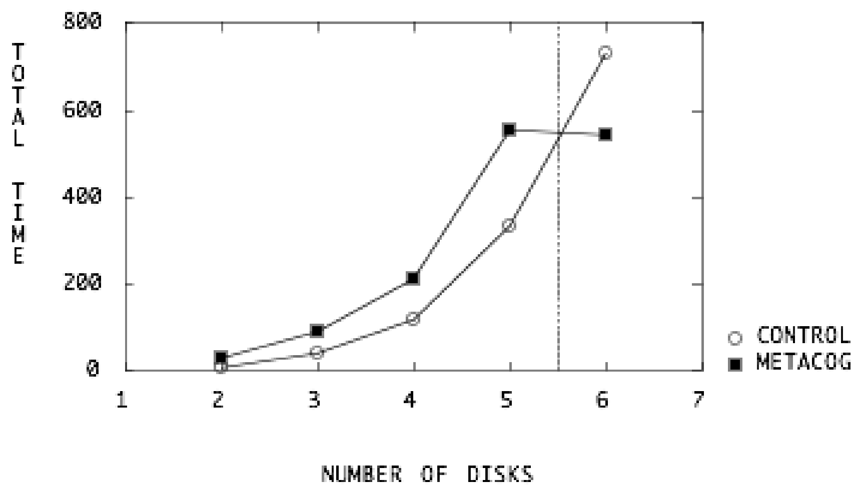

FIGURE 7: PERFORMANCE ACROSS 2- THROUGH 6-DISK VERSIONS OF THE TOWER-OF-HANOI PROBLEM (Time (sec) to Solution)

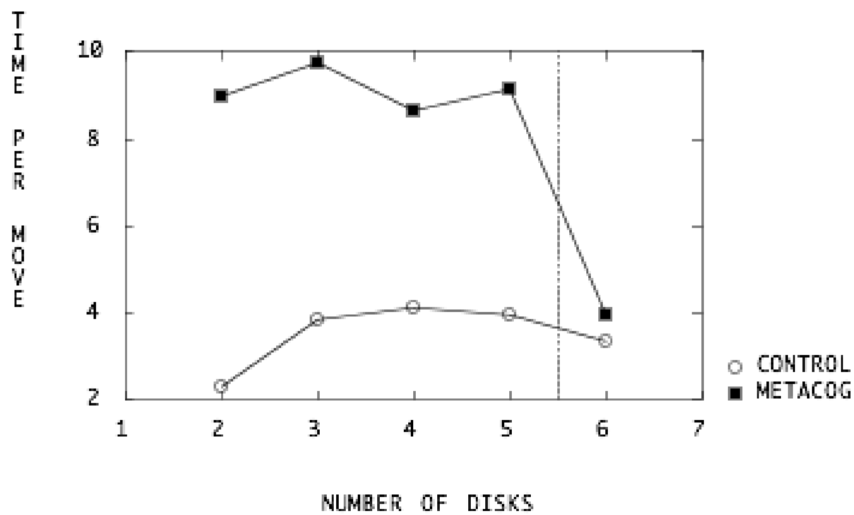

FIGURE 8: PERFORMANCE ACROSS 2- THROUGH 6-DISK VERSIONS OF THE TOWER-OF-HANOI PROBLEM (Time (sec) per move)

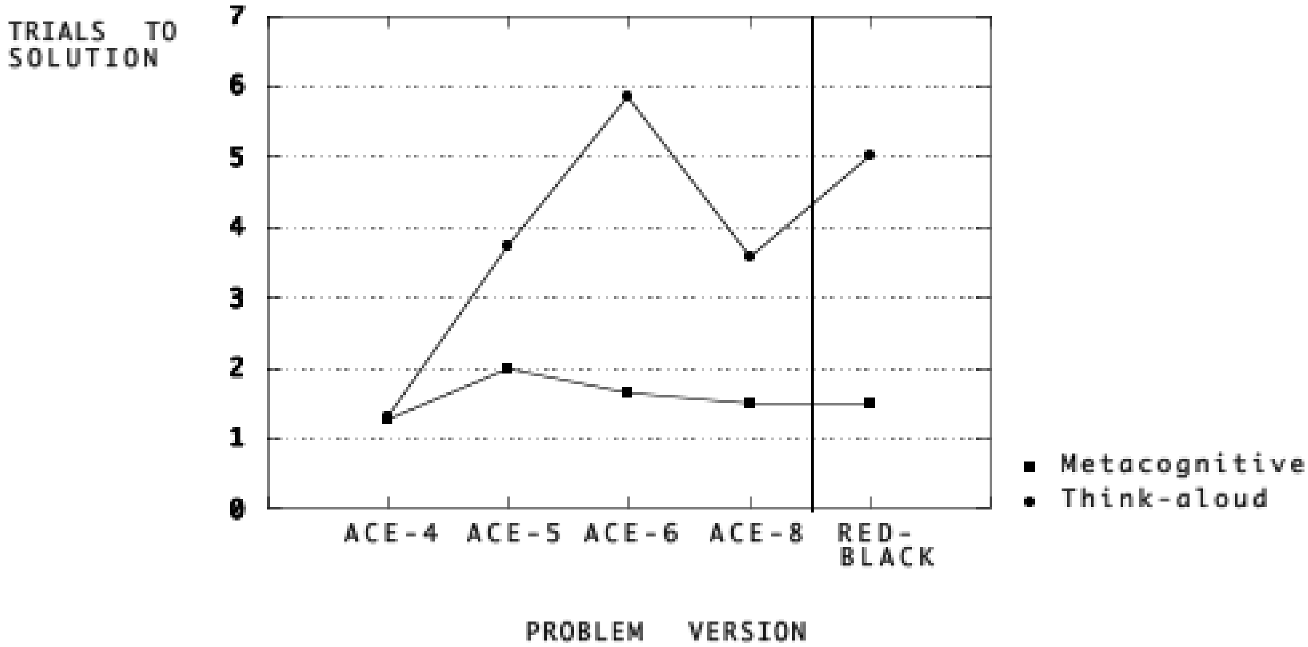

FIGURE 9: PERFORMANCE ACROSS THE 4 THROUGH 8 CARD VARIATIONS OF THE ACE, 2, 3, ... VERSION OF THE KATONA CARD PROBLEM AND ON THE RED/BLACK VERSION OF THE PROBLEM (solvers only)

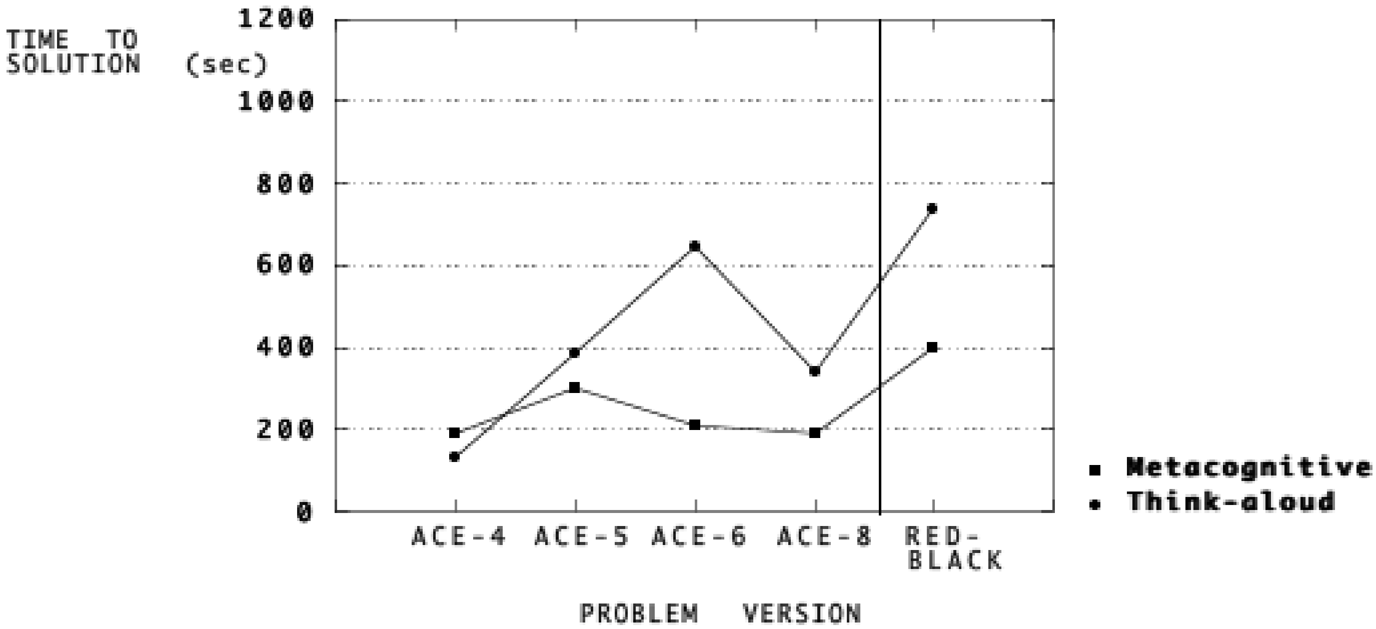

FIGURE 10: PERFORMANCE ACROSS THE 4 THROUGH 8 CARD VARIATIONS OF THE ACE, 2, 3, ... VERSION OF THE KATONA CARD PROBLEM AND ON THE RED/BLACK VERSION OF THE PROBLEM (solvers only)

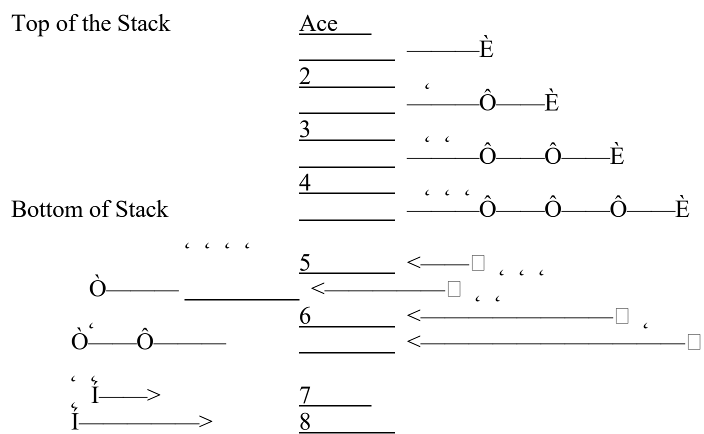

FIGURE 11: COMPLEX REPRESENTATION USED FOR DEALING-RULE IDENTIFICATION STRATEGY